LIMIT PROBLEMS ON HEAT OR MASS TRANSFER TO A CYLINDER AND SPHERE SUBMERGED IN AN INFILTRABLE GRANULAR LAYER

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UDC 536.24

The problems to be investigated below occur during the solution of problems on the cooling of contact reactors of chemical production, the diversion and subsequent utilization of heat from furnaces with granular layers by means of inserting different cooling bodies and inserts, within apparatus, in the thermal and diffusion treatment of articles in a fixed or slightly fluidized granular layer, in the problem of corrosion resistance of submerged inserts, etc. In the majority of situations of practical importance, the linear scale L of the body is much greater than the characteristic structural scale l of the layer, and in this case it is natural to use a continual description of the heat- or mass-transfer processes in a disperse mixture surrounding the body, by considering it as some homogeneous continuous medium (or as a system of several coexisting interacting media) with its own effective thermophysical and diffusion characteristics.

The macroscopic transfer equations in such media are derived either phenomenologically or on the basis of averaging local transfer equations, which are valid in the individual mixture phases, over a representative small physical volume or time interval [1-3], or over an ensemble of configurations of systems of disperse phase particles [4]. Equations from [4] are used below that are true when the direct heat or mass transfer of impurity over the contacts of contiguous particles is inessential. The assumption mentioned is always valid for mass transfer (including even a capillary-porous media) and is almost true for heat transfer in infiltrable granular layers. It is assumed that there are no possible heat or mass sources in the medium; this assumption is quite rough only when a catalytic reaction with a large thermal effect proceeding in an external diffusion or transition domain holds [5] and self-heating or self-cooling of the catalyst particles is essential because of the inadequate intensity of the interphasal heat transfer. The assumptions made permit description of the stationary transfer process within the framework of a single-phase dispersion model. Let us note that such a model can be used formally in a number of situations, even in cases when a two-phase model is necessary in the strict sense [6].

An assumption about the insignificant influence of the contact conduction and the inequality $L \gg l$ affords a possibility of neglecting in a first approximation the existence of a thin layer at the surface of the submerged body, whose mean properties differ from the properties of the dispersed medium far from the body.

§1. Statement of the Problem

Under the assumption mentioned above, the equation of stationary convective heat transfer in a mixture with a fixed dispersed phase can be written in the form [4]

$$C_0(\mathbf{u}\nabla)\tau = -\nabla\mathbf{q} - C_0\nabla\langle T'\mathbf{V}'\rangle, \qquad (1.1)$$

where C_0 is the specific heat of unit volume of the fluid being filtered; $\mathbf{u} = \varepsilon\mathbf{v}$ is the rate of filtration; the prime denotes fluctuation in the temperature T and the true field velocity $\mathbf{V}$ in the gaps between particles relative to their mean values τ and $\mathbf{v}$ (the layer porosity ε is considered independent of the coordinates).

The quantity $\mathbf{q}$ is the mean heat flux in the system which is not related to the fluid fluctuations,

$$\mathbf{q} = -\lambda\nabla\tau,\ \lambda = \lambda_0 F(\varepsilon,\ \lambda_1/\lambda_0), \qquad (1.2)$$

where λ is the effective thermal conductivity of the fluid-filled layer at $u = 0$, where there are theoretical (see [7], say) and numerous empirical [8] representations for the function F dependent on ε and on λ_1/λ_0 (the ratio between the thermal conductivities of the particle material and the fluid).

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki Tekhnicheskoi Fiziki, No. 5, pp. 94-102, September-October, 1977. Original article submitted June 21, 1976.

 0021-8944/77/1805-0666$07.50

The last member in the right side of (1.1) describes the convective heat dispersion in the intersected pore space of the granular layer. The tensor Λ^* of the corresponding effective coefficients of thermal conductivity has an axis of symmetry coincident in direction with the local rate of filtration [9], where the relationship [10]

$$C_0\nabla\langle T'\mathbf{V}'\rangle = \nabla(\Lambda^*\nabla\tau), \quad \lambda_i^* = 2k_iC_0lu \tag{1.3}$$

can be written for the principal values of this tensor.

Theoretical values of the coefficients of convective dispersion k_i are evaluated in [10] on the basis of a Markov model of uncorrelated sequential displacements (k_1 = 0.76, k_2 = 0.19) and agree fairly with experiment (k_1 = 0.7-0.8, k_2 = 0.1-0.3).

From (1.1)-(1.3) we obtain

$$C_0(\mathbf{u}\nabla)\tau = \nabla(\Lambda\nabla\tau), \quad \Lambda = \Lambda^* + \lambda\mathrm{I}, \tag{1.4}$$

where Λ is the tensor of the effective coefficients of thermal conductivity due to both molecular heat transfer and convective dispersion. (The presence of random local inhomogeneities in the granular layer results in the appearance of additional convective dispersion [11], which for simplicity is not taken into account below.

The rate of filtration is considered known from the solution of the appropriate hydrodynamic problem. Assuming the Darcy law valid (although in approximate form), we see that the distribution of u near the submerged body is actually obtained from an analysis of the problem of its potential flow.

Finally, we consider the temperature distribution or the normal heat-flux component given on the body surface; far from the body the temperature should agree with the temperature τ_∞ of the stream flowing in at a velocity $\mathbf{u}_\infty$ in the coordinate system coupled to the body.

In general respects, the problem formulation mentioned was discussed earlier in [12], where results referring to heat elimination from a cylinder and sphere whose surface was maintained at a constant temperature were also presented.

§2. Transformation of the Convective Heat-Conduction Equation

Since the tensor Λ is diagonal in coordinate systems with one of the axes directed along the local filtration rate, it is meaningful to analyze (1.4) in one such system. The system coupled to the isopotential surfaces φ = const and stream surfaces ψ = const introduced by Boussinesq is most natural for plane and axisymmetric meridian flows. The transformation to Boussinesq variables is also useful in the respect that it simplifies the convective part of (1.4) substantially since the inhomogeneous flow outside the body being streamlined is actually converted to a homogeneous flow in a plane with a slit.

Introducing the dimensionless variables and parameters

$$\begin{Bmatrix}\Phi\\ \Psi\end{Bmatrix} = \frac{1}{u_\infty L^{1+j}}\begin{Bmatrix}\varphi\\ \psi\end{Bmatrix}, \quad \mathrm{U} = \frac{u}{u_\infty}, \quad \rho = \frac{r}{L}, \qquad \mathrm{Pe} = \frac{C_0u_\infty L}{\lambda}, \quad \gamma_i = \frac{4k_iC_0u_\infty}{\lambda} \tag{2.1}$$

and taking account of (1.3) we obtain the equation

$$\frac{\partial\tau}{\partial\Phi} = \frac{1}{\mathrm{Pe}}\left\{\frac{\partial}{\partial\Phi}\left[\left(1+\frac{\gamma_1}{2}U\right)\frac{\partial\tau}{\partial\Phi}\right] + \frac{\partial}{\partial\Psi}\left[\left(1+\frac{\gamma_2}{2}U\right)(\rho\sin\theta)^{2j}\frac{\partial\tau}{\partial\Psi}\right]\right\} \tag{2.2}$$

in place of (1.4), where the values j = 0 and j = 1, respectively, correspond to the plane and axisymmetric flows in (2.1) and (2.2).

Equation (2.2) is equivalent to (1.4) in the domain U ≠ 0, i.e., everywhere with the exception of the stagnant frontal and root points on the body surface. Other equations could be obtained from (1.4) in place of (2.2) in a small neighborhood of these points and their solutions could then be merged with the solution of (2.2). However, such a procedure is physically meaningless in the case under consideration if the size of these singular domains is commensurate with or less than l, so that the dispersion model used, which results in (1.4), will itself become meaningless.

Therefore, (2.2) is analyzed in a domain with deleted stagnant points, and the boundary conditions should correspondingly be given on the body surface. The problem obtained can be considered as a Cauchy problem for the elliptic equation (2.2), which is incorrect in the classical (Hadamard) sense, but becomes correct upon narrowing the class of solutions because of the imposition of additional conditions (Tikhonov class of correctness). Boundary-value problems of this kind for the Helmholtz equation, to which (2.2) can be reduced, have been considered in [13], where it has been shown that the boundedness condition

$$\sum_{i=1,2} \lim_{\sigma_i \to 0} \oint_{c_i(\sigma_i)} \left\{ |\tau| + \left| \frac{\partial \tau}{\partial \sigma_i} \right| \right\} dc_i = 0$$

must be used as the additional condition assuring uniqueness of the solution of problems with an open boundary, where the summation is over the two singular (stagnant) points and the integration is over circular contours of small radius around these points. In substance, this condition requires the absence of nonintegrable singularities at the points mentioned.

The solution of (2.2) for sufficiently general conditions on the body surface is quite problematical. Only the exact solution of the second boundary-value problem for a cylinder [14] and the possibility of reducing the solution of the plane first boundary-value problem [15] to the solution of an analogous problem for a plate [16] if there is no convective dispersion are known. In the general case it is expedient to use the method of perturbations [17], associated with the construction of a decomposition of the desired solution in some system of congruent functions $\{\delta_n(\mathrm{Pe})\}$ which tends to zero as Pe → 0 or Pe → ∞. These limit decompositions can be convergent for Pe → ∞ (when they converge to the exact solutions) or divergent (when they are asymptotic expansions of the solutions). We deal with problems about singular perturbations in both limit situations. As Pe → ∞ this is associated with the fact that there is a small parameter in (2.2) in the highest derivatives. As Pe → 0, the singular nature of the perturbations is due to the fact that the asymptotic expansion is nonuniform in the sense of [17] in the neighborhood of the infinitely remote point and there is the need to construct an additional internal asymptotic expansion, which would then merge with the external expansion.

§3. Solution of the Problem for Pe << 1

In this case the members describing the convective dispersion in (2.2) contain two small factors Pe and l/L and can be omitted so that the tensor Λ is global and its eigenvalue equals λ.

Let us first examine the problem of a cylinder when (1.4) can be written in the form

$$\mathrm{Pe}(\mathbf{U}\nabla)\tau = \nabla^2\tau, \tag{3.1}$$

where the operator ∇ is defined in the space **ρ** of dimensionless coordinates. For large ρ we introduce the internal variables

$$\mathbf{R} = \mathrm{Pe}\,\boldsymbol{\rho},\ X = R\cos\theta,\ Y = R\sin\theta \tag{3.2}$$

in the usual manner.

Using (3.2) in (3.1), we obtain an equation for τ in the variables R and θ (or X and Y) which contains Pe as a small parameter. It is natural to seek the solution in the form

$$\tau = \sum_{n=0}^{\infty} \tau^{(n)}\delta_n(\mathrm{Pe}),\quad \delta_0 = 1,\quad \lim_{\mathrm{Pe}\to 0} \delta_n(\mathrm{Pe}) = 0. \tag{3.3}$$

Limiting ourselves to the first member of the series (3.3) we obtain an equation of convective transfer in a homogeneous stream for $\tau^{(0)}$,

$$\frac{\partial \tau^{(0)}}{\partial X} = \frac{\partial^2 \tau^{(0)}}{\partial X^2} + \frac{\partial^2 \tau^{(0)}}{\partial Y^2}, \tag{3.4}$$

whose bounded solution, which satisfies the known radiation condition and is written in the ρ and θ variables, has the form

$$\tau^{(0)} = \tau_\infty + C_1 \exp\left(\frac{\mathrm{Pe}}{2}\rho\cos\theta\right) K_0\left(\frac{\mathrm{Pe}}{2}\rho\right), \tag{3.5}$$

where $K_0(x)$ is the Macdonald function.

The monomial external expansion is determined as the solution of the Laplace equation which follows from (3.1) as Pe → 0. For the Dirichlet problem [the temperature distribution on the cylinder surface $\tau_0(\theta)$ is given] this solution is written in the form

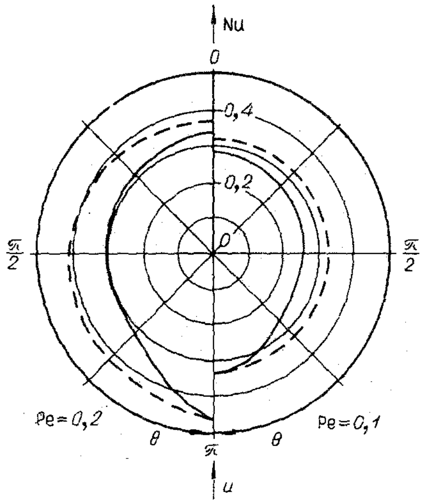

Fig. 1

$$\tau^{(0)} = \frac{a_0}{2} + \sum_{i=1}^{\infty}\left(\frac{1}{\rho}\right)^i (a_i \cos i\theta + b_i \sin i\theta) + C_2 \ln\frac{1}{\rho}, \tag{3.6}$$

where a_i and b_i are Fourier coefficients of the function $\tau_0(\theta)$.

For the Neumann problem [the density distribution of the normal heat flux component $Q(\theta)$ is given], we have

$$\tau^{(0)} = -\frac{A_0}{2}\ln\frac{1}{\rho} - \sum_{i=1}^{\infty}\frac{1}{i}\left(\frac{1}{\rho}\right)^i (A_i \cos i\theta + B_i \sin i\theta) + C_3, \tag{3.7}$$

where A_i and B_i are the Fourier coefficients of the function $Q(\theta)$.

The arbitrary constants C_1 and C_2 or C_1 and C_3 in (3.5)-(3.7) are determined from the condition or merger of the asymptotic expansions. By using the asymptotic representation of the Macdonald function, we obtain for the first boundary-value problem

$$C_1 = C_2 = \left(\frac{a_0}{2} - \tau_\infty\right)\ln^{-1}\frac{4}{\gamma P} \tag{3.8}$$

and for the second boundary-value problem

$$C_1 = \frac{A_0}{2}, \quad C_3 = \frac{A_0}{2}\ln\frac{4}{\gamma \mathrm{Pe}} + \tau_\infty, \tag{3.9}$$

where γ is the Euler constant. Finally, (3.8) and (3.9) determine the solution of both problems which can be represented, in the usual way, in the form of composite asymptotic expansions

$$\tau^{(0)} = \tau_\infty + \left(\frac{a_0}{2} - \tau_\infty\right)\ln^{-1}\frac{4}{\gamma \mathrm{Pe}}\exp\left(\frac{\mathrm{Pe}}{2}\rho\cos\theta\right)K_0\left(\frac{\mathrm{Pe}}{2}\rho\right) + \sum_{i=1}^{\infty}\left(\frac{1}{\rho}\right)^i (a_i \cos i\theta + b_i \sin i\theta); \tag{3.10}$$

$$\tau^{(0)} = \tau_\infty - \frac{A_0}{2}\exp\left(\frac{\mathrm{Pe}}{2}\rho\cos\theta\right)K_0\left(\frac{\mathrm{Pe}}{2}\rho\right) - \sum_{i=1}^{\infty}\frac{1}{i}\left(\frac{1}{\rho}\right)^i (A_i \cos i\theta + B_i \sin i\theta). \tag{3.11}$$

A conception of the accuracy of the asymptotic solutions (3.10) and (3.11) can be obtained from Fig. 1, where we compare the local values of the Nusselt number at different points on the cylinder surface, which follows from the exact solution of the second boundary-value problem in and from (3.11) (solid and dashed curves, respectively) for two values of the Peclet number for a constant heat-flux density on the cylinder surface (Q = const); the arrow shows the freestream direction.

Let us now consider the analogous problem for a sphere. As before, the main equation has the form (3.1), the internal variables are introduced analogously to (3.2), and the solution is also expanded in the form of the series (3.3). Solving the equation for the first member of this series, which replaces (3.4) in this case, we obtain in place of (3.5)

$$\tau^{(0)} = \tau_\infty + \frac{C_1}{\rho \mathrm{Pe}} \exp\left[\frac{\mathrm{Pe}}{2} \rho (1 - \cos\theta)\right]. \tag{3.12}$$

After evaluation, we obtain in place of (3.6) and (3.7), respectively,

$$\tau^{(0)} = \sum_{i=0}^{\infty} \frac{1}{\rho^{i+1}} \sum_{m=1}^{M} a_{im} Y_{im}(\theta) + C_2\left(1 - \frac{1}{\rho}\right), \quad M = 2i + 1; \tag{3.13}$$

$$\tau^{(0)} = -\frac{A_{00}}{\rho} - \sum_{i=1}^{\infty} \frac{1}{(1+i)\rho^{i+1}} \sum_{m=1}^{M} A_{im} Y_{im}(\theta) + C_3, \tag{3.14}$$

where $Y_{im}(\theta)$ is the m-th spherical function of i-th order, and a_{im} and A_{im} are coefficients of the expansions of the functions $\tau_0(\theta)$ and $Q(\theta)$ in spherical functions.

As before, merging the internal expansion (3.12) with the external expansion (3.13) or (3.14) permits finding the arbitrary constants in (3.12)-(3.14). The appropriate composite asymptotic expansions have the form

$$\tau^{(0)} = \sum_{i=0}^{\infty} \frac{1}{\rho^{i+1}} \sum_{m=1}^{M} a_{im} Y_{im}(\theta) + \tau_\infty \left\{1 - \frac{1}{\rho} \exp\left[-\frac{\mathrm{Pe}}{2} \rho (1 - \cos\theta)\right]\right\};$$

$$\tau^{(0)} = -\frac{A_{00}}{\rho} \exp\left[-\frac{\mathrm{Pe}}{2} \rho (1 - \cos\theta)\right] - \sum_{i=1}^{\infty} \frac{1}{(1+i)\rho^{i+1}} \sum_{m=1}^{M} A_{im} Y_{im}(\theta) + \tau_\infty,$$

where the parameter M is defined in (3.13).

§4. Solution of Problems for $\mathrm{Pe} \to \infty$

In this case the convective heat dispersion can be quite substantial (the tensor Λ differs from the global tensor) and it is convenient to use (2.2). By using the Poincaré–Lighthill–Ho method [17] and introducing the small parameter $\varepsilon = \mathrm{Pe}^{-1/2}$, we represent the desired function τ and the independent variables Φ and Ψ in the form

$$\tau = \sum_{i=0}^{\infty} \varepsilon^i \tau^{(i)}, \quad \Phi = \xi + \sum_{i=1}^{\infty} \varepsilon^i \Phi_i(\xi, \eta), \quad \Psi = \varepsilon\left[\eta + \sum_{i=1}^{\infty} \varepsilon^i \Psi_{i+1}(\xi, \eta)\right]. \tag{4.1}$$

The transformation from Φ, Ψ to ξ, η in (4.1) is the result of the combined utilization of perturbed coordinates and boundary-layer methods. In fact, these transformations correspond not only to the shear strain of the coordinate grid, but also to the typical stretching of the "transverse" coordinate Ψ in the boundary layer. Success in applying (4.1) is hence usually related to how successfully the variables ξ, η have been chosen.

The quantities Φ and Ψ can be represented as follows:

$$\Phi = \left[\rho + \frac{1}{(1+j)\rho^{1+j}}\right] \cos\theta, \quad \Psi = \frac{1}{1+j}\left(\rho^{1+j} - \frac{1}{\rho}\right)(\sin\theta)^{1+j},$$

where as before j = 0 for a cylinder and j = 1 for a sphere. By defining ξ and η in the form

$$\xi = \frac{2+j}{1+j} \cos\theta, \quad \eta = \frac{2+j}{1+j} \frac{\rho - 1}{\varepsilon} (\sin\theta)^{1+j},$$

we obtain the following expansion of the type (4.1):

$$\Phi = \xi + \varepsilon^2 \xi \eta^2 \frac{(1+j)^3}{2(2+j)^2} \left[1 - \frac{(1+j)^2}{(2+j)^2} \xi^2\right]^{-1-j} + \ldots, \quad \Psi = \varepsilon\eta + \varepsilon^2 \xi \eta^2 \frac{(j^2 + j - 2)(1+j)}{2(2+j)^2} \left[1 - \frac{(1+j)^2}{(2+j)^2} \xi^2\right]^{-(1/2)(1+j)} + \ldots \tag{4.2}$$

The operators of differentiation with respect to Φ and Ψ are expressed by standard means in the form of linear combinations of differentiation operators with respect to ξ and η, where the coefficients in these combinations ($\partial\xi/\partial\Phi$ etc.) are evaluated from a system of linear equations obtained because of differentiating (4.2) with respect to Φ and Ψ.

Using the mentioned operator equations, the obvious relations

$$U = |\nabla\Phi| = \frac{2+j}{1+j}\sin\theta + O(\varepsilon), \quad (\rho\sin\theta)^{2j} = \sin^{2j}\theta + O(\varepsilon),$$

and the perturbation method, we have from (2.2) for the first member of the series (4.1) after calculations

$$\frac{\partial\tau^{(0)}}{\partial\xi} = \left[1 - \left(\frac{1+j}{2+j}\xi\right)^2\right]^j \left\{1 + \frac{\gamma_2}{2}\,\frac{2+j}{1+j}\left[1 - \left(\frac{1+j}{2+j}\xi\right)^2\right]^{1/2}\right\}\frac{\partial^2\tau^{(0)}}{\partial\eta^2}. \tag{4.3}$$

Introducing the new variable

$$t = \int\limits_{-(2+j)/(1+j)}^{\xi} \left[1 - \left(\frac{1+j}{2+j}\xi\right)^2\right]^j \left\{1 + \frac{\gamma_2}{2}\,\frac{2+j}{1+j}\left[1 - \left(\frac{1+j}{2+j}\xi\right)^2\right]^{1/2}\right\} d\xi = \frac{2+j}{1+j}\int\limits_{\theta}^{\pi} (\sin\theta)^{1+2j}\left(1 + \frac{\gamma_2}{2}\,\frac{2+j}{1+j}\sin\theta\right) d\theta, \tag{4.4}$$

we obtain a simple parabolic equation from (4.3),

$$\partial\tau^{(0)}/\partial t = \partial^2\tau^{(0)}/\partial\eta^2, \tag{4.5}$$

whose solution should satisfy some initial condition at $t = 0$ (i.e., at the point $\theta = \pi$ of the incoming stream, in addition to the boundary condition at the body surface and the condition at infinity). The condition $\tau_0(t = 0) = \tau_\infty$ is ordinarily used.

As a result of the solution of (4.5) under the mentioned conditions, we obtain the following asymptotic representation for the solution of the first boundary-value problem:

$$\tau^{(0)} = \tau_\infty \operatorname{erf}\frac{\eta}{2\sqrt{t}} + \frac{1}{2\sqrt{\pi}}\int\limits_0^t \frac{\eta\tau_0(t')}{(t-t')^{3/2}}\exp\left[-\frac{\eta^2}{4(t-t')}\right]dt'. \tag{4.6}$$

For the second boundary-value problem, we have instead of (4.6)

$$\tau^{(0)} = \tau_\infty - \frac{1}{\sqrt{\pi}}\int\limits_0^t \frac{Q(t')}{(t-t')^{1/2}}\exp\left[-\frac{\eta^2}{4(t-t')}\right]dt'.$$

The variables t and η are defined by the relations presented above and $\tau_0(t)$ and $Q(t)$ are functions defined in terms of values of the temperature and the normal heat-flux density component given on the body surface after they have been substituted into the last values of the argument θ expressed in terms of t in conformity with (4.4)

Distributions of the ratio between the local and the mean values of the Nusselt number over the surface are presented in Fig. 2a and b for an isothermal cylinder and an isothermal sphere, respectively, for different values of γ_2. It is seen that the presence of convective heat dispersion results in a noticeable change in these distributions as compared to the distributions characteristic for the situations where there is only molecular transfer ($\gamma_2 = 0$).

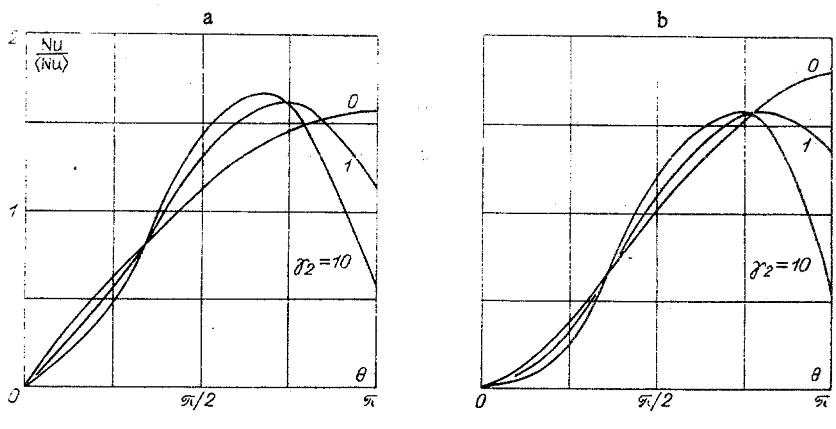

Fig. 2

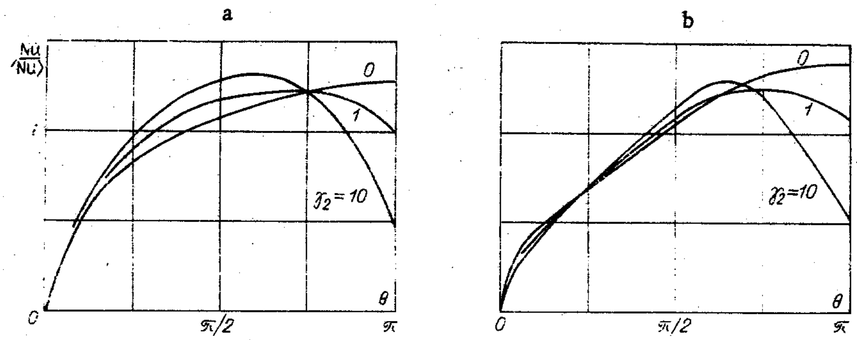

Fig. 3

In particular, the maximum heat elimination is shifted from the domain of the incoming stream down along the streamlined surface where the local filtration rate (and therefore, the coefficient of transverse convective dispersion also) is higher. Analogous curves are presented in Fig. 3a and b for a cylinder and sphere, respectively, under the condition that a uniformly distributed heat flux is given on their surfaces (the notation is the same as in Fig. 2).

Let us note that results analogous to those presented in Fig. 2 and 3 for $\gamma_2 = 0$ have been obtained earlier in [18, 19] in application to problems on the heat transfer from a cylinder and sphere with fluids characterized by low Prandtl numbers, when the applicability of the potential flow model is due to the fact that the dynamic boundary layer is embedded in the thermal layer so that this latter lies principally in the inviscid flow domain. Satisfactory correspondence between the theoretical results from [18] and the experimental data from [20] indicates indirectly the adequacy of the theory developed above even for $\gamma_2 \neq 0$.

In conclusion, let us note the fundamental conditions for applicability of the theory: smallness of the structural scale of the granular layer as compared to the body characteristic dimension, insignificant influence of the contact conductivity on the total heat flux in the granular layer, and absence of substantial heat sources and sinks. Moreover, the validity of the results obtained in the limit case $Pe \to \infty$ is spoiled for very large Pe when the thermal boundary-layer thickness becomes commensurate with l and it is necessary to take account of the difference in the properties of the thin near-surface layer and the effective properties of the granular medium far from the body even in the case when $L \gg l$ and there is not contact heat transfer.

We spoke above only about heat transfer, but all the results obtained are equally valid for the mass transfer of an impurity.

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EFFECT OF THE AXIAL COMPONENT OF THE HEAT FLUX ON SOLIDIFCATION OF A METAL WITH CONTINUOUS CASTING

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UDC 669-147

In many pieces of work devoted to the theory of the solidification of a metal with continuous casting, as a rule, the axial component of the heat flux is neglected and an approximate equation of thermal conductivity with constant thermophysical parameters of the metal is considered [1-5]. The present article, on the basis of an exact equation of the thermal conductivity, considers the process of the solidification of a continuous ingot, with an arbitrary dependence of the thermophysical parameters of the metal on the temperature. From an analysis of the self-similar solution found, a condition is obtained, with which an approximate consideration of the problem without taking account of the axial component of the heat flux is valid. As an example, let us examine the process of the solidification of a flat aluminum ingot.

We shall postulate that a flat ingot with a thickness of $2x_0$ moves along the Z axis with a constant velocity v. Here we assume that the temperature of the melt (the liquid phase) is equal to the crystallization temperature T_{cr}.

The equation determining the distribution of the temperature T in the solid phase under fully established conditions has the form

$$vc_V\frac{\partial T}{\partial z}=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right), \quad (1)$$

where the volumetric heat capacity c_V and the thermal conductivity λ depend on the temperature T.

We write the boundary condition at the cooled surface of the ingot in the form

$$\lambda\frac{\partial T}{\partial x}\Big|_{x=x_0}=q(z), \quad (2)$$

where q(z) is the law of heat removal, whose form will be determined below. Specifically, if the heat removal takes place according to the Newton—Rajchman law. Then, we set

$$q(z)=k(z)\left[T|_{x=x_0}-T_c(z)\right]. \quad (3)$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 102-107, September-October, 1977. Original article submitted October 12, 1976.

0021-8944/77/1805-0673$07.50